AN INVESTIGATION INTO THE INTERACTION OF A STRESS WAVE WITH A STATIONARY MACROCRACK IN ELASTIC-PLASTIC AND QUASIBRITTLE MATERIALS

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A large number of publications are devoted to the behavior of materials under dynamic loads. For example, the effect of the strain rate on the mechanical properties of metals [1, 2], the delay of yield under short-duration loads [3], the dependence of stresses on the strain rate [4,5], as well as the effect of dynamic loading on the statistical properties of metals have been investigated.

In this work we present certain results of an investigation, by the polarized-light method in combination with high-speed cinematography, into the stress field in the vicinity of a stationary macrocrack subjected to a stress wave.

The investigation is carried out on polymers. The diffraction phenomenon of the pressure wave at the tip of a stationary macrocrack is established experimentally. The diffracting wave propagates against the original wave from the opposite bank of the crack. The stress concentration is determined by the angle of entry of the wave into the plane of crack. The largest stress concentration occurs for minimum angles of incidence. This means that displacements of particles of the media directed along the crack play the major role in the formation of the stress field at the tip of the crack.

1. Description of Experiments. The diagram of the installation used for experiments is shown in Fig. 1. In this diagram: 1, pulsed light source; 2, condensing lens; 3, polarizer; 4, quarter wave plate; 5, testpiece; 6, supply electrodes; 7, tube with water; 8, quarter wave plate; 9, analyzer; 10 lens of cine

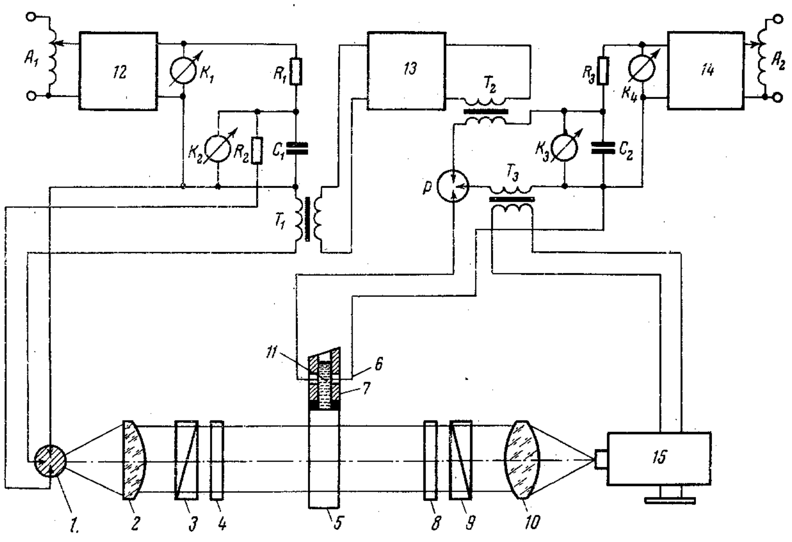

Fig. 1

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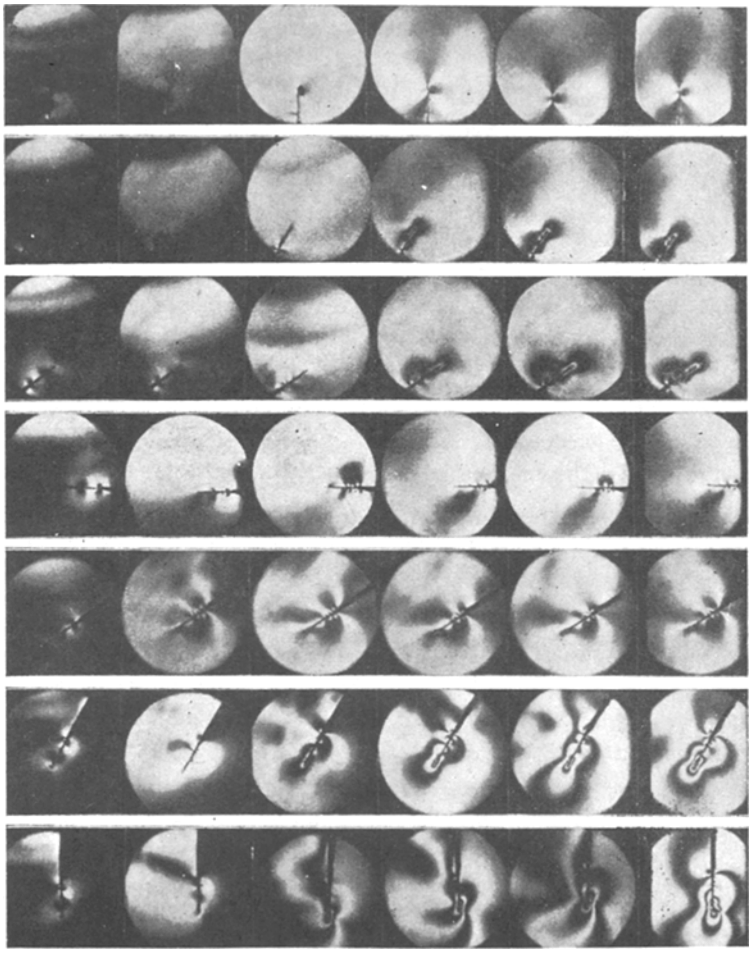

Fig. 2

camera; 11, breaking wire; 12, high-voltage rectifier; 13, delay line; 14, high-voltage rectifier; 15, cine camera SFR-1. Capacitors are charged by means of two high-voltage rectifiers via the charging resistors R_1 and R_3. The value of the charging voltage is regulated by autotransformers from the instruments K_1 and K_4. After charging the capacitors C_1 and C_2 up to the source voltage (control is effected by means of the kilovoltmeters K_2 and K_3), the high-speed SFR-1 cine camera is switched on. The discharger P is fired by a high-voltage pulse fed from the control desk. Here discharge of the capacitor C_2 takes place through a wire with a diameter of 0.02 mm. As a result the wire breaks, and a shock wave through a column of liquid is transmitted to the testpiece. By means of the transformer T_2, which has the supply line of the breaking wire as its primary winding, simultaneously with the break a pulse is fed through the delay line to the high-voltage transformer T_1,which initiates the operation of a powerful light source. The delay line effects synchronization of operation of the tube L_1 with the instant when the wave proceeds into part of the testpiece which is to be cine-filmed. In the role of a pulsed light source an ISSh-100-3 tube, based in the focus of the condensing lens, is used. The small dimensions of its luminous element enable us to obtain a practically parallel pencil of rays. The flash energy is more than 3000 J; its duration is 200-250 μsec.

The testpieces were made from Plexiglas in the form of circular discs having a diameter of 180 mm and thickness of 18 mm. Then incisions were made on the testpieces; a crack was produced by a slight impact by a knife at the vertex of these incisions. To remove the internal stresses, the testpieces were annealed for 5-6 days and nights at a temperature of 120°C with subsequent slow cooling at a rate of 5°C per h.

2. Analysis of Experiments. The testpieces thus prepared were subjected to impulsive loading by a pressure wave. The possibility of orienting the crack relative to the direction of motion of the wave enabled us to experimentally investigate the stress field at the vertex of the crack for various angles of attack of the wave. The frames of cine film of the interaction of the wave and the crack, obtained at the rate of taking 480,000 frames per sec, are presented in Fig. 2. An analysis of them shows that,from the instant

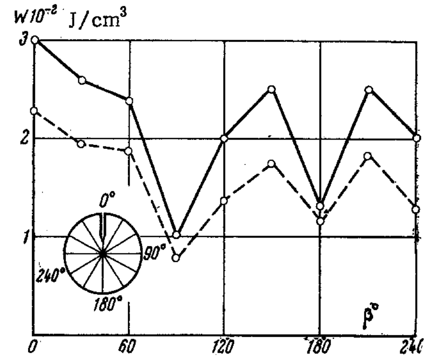

Fig. 3

when the wave goes out to the bank of the crack, a field of dynamic stresses is gradually formed at its vertex. The value of stresses depends on the angle of incidence of the wave. The value of energy W stored in the neighborhood of the crack tip, dependent on the angle of incidence of the wave, is shown in Fig. 3. The value in a constant volume was computed from the expression

$$W = \frac{(\tau_0^{1.0})^2}{2tE} \sum_{i=0}^{i=n} n_i^2 s_i \tag{2.1}$$

where t is the thickness of the testpiece, n is the fringe number, $\tau_0^{1.0}$ is the fringe value under dynamic loading, E is the modulus elasticity, and s is the area of the fringe.

(The calculations were carried out for the time 75 μsec, measured from the instant when the wave went out to the vertex of the crack; the frames before fracture correspond to this time.)

From the graph we see that the maximum value of the energy and the maximum stress concentration arise when the wave propagates to only one of the banks of the crack ($\beta = 0$). In this case the stress concentration reaches a value which exceeds the ultimate strength; as a result, growth of the crack takes place at an angle of 80-85° to its original direction.

When the wave falls at an angle to the plane of the crack, the stresses at its vertex decrease, assuming the minimum value at $\beta = 90°$. It should be noted that when the wave travels along the crack, but in the opposite direction ($\beta = 180°$), the same concentration does not occur as in the case where the directions of their motions coincide, i.e., when $\beta = 0$. Apparently this is explained by a uniform energy distribution between the banks of the crack and, therefore, the stress rosette is symmetric and weak.

If the wave is directed at an angle of 30° or 120°, considerable stress concentrations occur at the vertex of the crack regardless of its direction.

Thus, important and dangerous in relation to the fracture will be the case where the wave propagates along one of the banks of the crack. For this case experiments were carried out on steels, and an analysis was conducted into the distribution of the maximum shear stresses obtained on Plexiglas.

Steels 65G and ShKh-15 of standard composition were investigated. In order to obtain a brittle state, testpieces in the form of $300 \times 300 \times 10$ mm^3 plates with an incision were subjected to hardening (850-860°C) in hot oil (the temperature of the oil was 70°C) with a subsequent tempering at T = 180°C for two hours. As a result of the thermal stresses hardening cracks (l = 7-10 mm) were formed at the tip of the incision. When an impact was applied to one of its sides, the embryonic cracks changed their direction of propagation [in Fig. 4 the specimen fractures of Plexiglas (a) and steel (b) are shown]. It is characteristic that the angle of the crack grown in steel changed in the same way as in Plexiglas (i.e., forming at 80-85° to the original direction).

From the data just presented it follows that when the wave diffracts on the crack, dynamic stresses are created in its vicinity. These stresses can initiate the growth of the crack; at the same time, independently of the material and its structure, the fracture takes place in a strictly defined direction.

To explain the change in the direction of propagation of the crack when it interacts with a stress wave, we solved a plane problem of the dynamic theory of elasticity concerning stress waves which arise in an infinite plate with an angular cutout (similar to a crack; Fig. 5) when an elemental plane longitudinal wave of the form

$$\begin{gathered} \varphi = s^\circ\left(t - \theta_0 x + \sqrt{a^{-2} - \theta_0^2 y}\right) \qquad (a\theta_0 = \arccos\beta) \\ s^\circ(\xi) = 1 \text{ when } \xi > 0, \qquad s^\circ(\xi) = 0 \text{ when } \xi = 0 \end{gathered} \tag{2.2}$$

falls at the tip of the cutout at the instant t = 0.

Here a is the velocity of the longitudinal wave, and θ_0 is a constant which gives the direction of incidence of the wave.

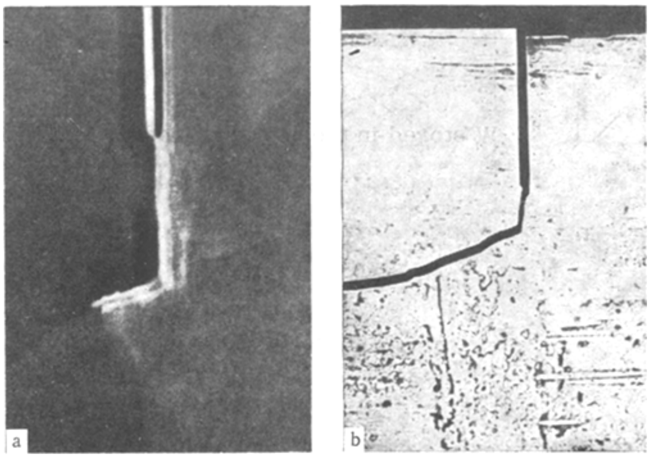

Fig. 4

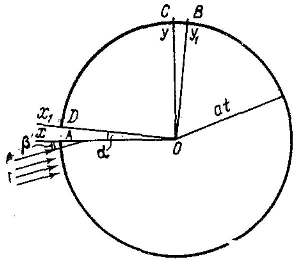

Fig. 5

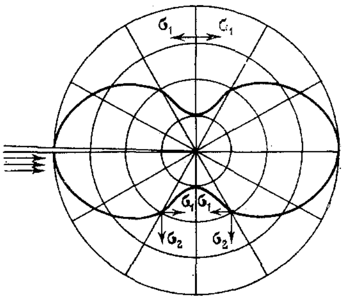

Fig. 6

The wave gives rise to a diffractive disturbance which at the time instant t fills a sector of radius at and angle $2\pi - \alpha$, and is described by the longitudinal potential φ and the transverse potential ψ. In the solution of this problem, it was assumed that a plane wave at small angles of incidence does not give rise to a displacement of particles on the boundary in a direction perpendicular to the surface of the crack. That is to say, at the incidence of the longitudinal wave the reflected transverse wave is absent. Therefore, the following boundary conditions were taken:

$$\tau_{xy} = 0,\ v = 0 \text{ when } y = 0 \qquad \tau_{x'y'} = 0,\ v' = 0 \text{ when } y' = 0$$

It is natural to assume that the diffractive disturbance within the sector OABCDO (Fig. 5) is solely longitudinal, i.e., we take $\psi = 0$. Then the problem reduces to finding within the sector OABCDO a longitudinal potential φ which, within the region under consideration, satisfies the wave equation

$$\frac{d^2\varphi}{dx^2} + \frac{d^2 y}{dy^2} = \frac{1}{a}\frac{d^2\varphi}{dt^2} \tag{2.3}$$

while on the sides of the angle AOD it satisfies the equation

$$v = \frac{d\varphi}{dy} = 0 \tag{2.4}$$

and on the sector OABCD part of the circumference it satisfies the equation $\varphi = \text{const}$.

The boundary conditions $\tau_{xy} = 0$ for y = 0 are satisfied automatically when the conditions (2.4) are fulfilled. Indeed,

$$\tau_{xy} = 2\mu\,\frac{d^2\varphi}{dx\,dy} = 2\mu\,\frac{dv}{dx}$$

Therefore, $\tau_{xy} = 0$ for v = 0.

The solution of this problem in terms of elementary functions was obtained in [7]. The following expression was presented in [8] for the functions

$$\varphi(x, y, t) = \operatorname{Re}\frac{1}{\pi i}\ln\frac{(e^{-\gamma_1 i} - z)(e^{-\gamma_2 i} - z)}{(e^{\gamma_1 i} - z)(e^{\gamma_2 i} - z)} + \frac{\gamma_1 + \gamma_2}{\pi}$$

$$z = (e^{-\varkappa i j})^n, \qquad j = \frac{at}{r} - \left(\frac{a^2t^2}{r^2} - 1\right)^{1/2} e^{\theta i}$$

Here r and θ are the polar coordinates of points within the sector OABCDO

$$\gamma_1 = \frac{\pi(\beta - \alpha)}{2\pi - \alpha}, \qquad \gamma_2 = \pi - \frac{\pi\beta}{2\pi - \alpha}, \qquad n = \frac{\pi}{2\pi - \alpha}$$

To investigate the stress distribution within the region under consideration the components of the stress state were determined from the following expressions:

$$\sigma_x = 2\mu \frac{d^2\varphi}{dx^2} + \lambda\left(\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2}\right)$$
$$\sigma_y = 2\mu \frac{d^2\varphi}{dy^2} + \lambda\left(\frac{d^2\varphi}{dx^2} + \frac{d^2\varphi}{dy^2}\right)$$
$$\tau_{xy} = 2\mu \frac{d^2\varphi}{dxdy}$$

The principal stresses, maximum shear stresses, and the angles of inclinations of the principal stresses to the coordinate axes were found from the expressions

$$\sigma_{1,2} = {}^1/_2(\sigma_x + \sigma_y) \pm \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$
$$\tau_{\max} = {}^1/_2 \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$
$$\theta_{1,2} = \frac{1}{2} \operatorname{arc\,tg} \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

The quantities σ_1, σ_2, $\tau_{\max}$, and $\theta_{1,2}$ were computed by means of an M-20 computer for the values

$$\alpha = 1^\circ, \quad \beta = 0,\ 2,\ 3,\ 5^\circ$$

The following values were taken for the Lamé constants (Plexiglas):

$$\mu = 1.96 \cdot 10^{10}, \quad \lambda = 4.54 \cdot 10^{10}$$

When calculating the principal and maximum shear stresses, the value of r was varied from 1 to 30 mm in steps of 5 mm. The angle θ was varied from 0 to $2\pi - \alpha$ in steps of 10°, and *at* varies from 0 to 240° in steps of 20 μsec. The curves of equal shear stresses were constructed from the results of the calculations (Fig. 6).

A comparison of the calculated data with the experimental results showed that, for small angles of incidence $\beta \leq 30°$ of the wave, the patterns of isochromatics obtained by means of cinematography coincide with the curves plotted for equal shear stresses. This enables us to draw the conclusion that the stress diagrams plotted from the calculation results correspond to the actual stresses which arise on the sides of the crack.

Calculations of the angle θ, which characterizes the inclinations of the principal stresses to the coordinate axes, show that it amounts to several degrees. Consequently, one of the principal stresses coincides with the AO axis, which is parallel to the surface of the main crack.

As is seen from Figs. 4-6, a concentration of tensile stresses is observed at the tip of the crack. At the same time the maximum stresses arise in the plane which is perpendicular to the surface of the crack. This result is well confirmed by the pattern of crack formation during experiments (Fig. 4).

Analyzing the theoretical and experimental data, we can draw the conclusion that, in the process of stress formation at the crack tip, displacements of particles of the material directed along the crack are of decisive value. As the angle of entry of the wave increases, the component of longitudinal displacements decreases and, accordingly, the stress concentration at the crack tip decreases.

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